INFORMATION ON EXPECTATIONS ABOUT THE ESCUDO CONVERGENCE FROM THE VOLATILITY IMPLIED IN CURRENCY OPTIONS*

Bernardino Adão**
Nuno Cassola**
Jorge Barros Luis**

1. INTRODUCTION

On 1 January 1999 a new international currency will be born — the euro — whilst the national currencies of the European Union Member-states participating in the Third Stage of the Economic and Monetary Union (EU-11) will cease to exist. Reflecting the European Union Treaty and other legal dispositions relative to the introduction of the euro, the conversion rates to the euro of the participating currencies will only be known on the last market day, in December 1998. However, as announced on 3 May 1998, the current bilateral central parities of the Exchange Rate Mechanism of the European Monetary System (ERM-EMS) will be used in the calculation of the irrevocable conversion rates to the euro. Admitting that the announcement of the participating countries and the rule of conversion to euro are credible, several implications exist to the behaviour of the interest and exchange rates in the transition to the Economic and Monetary Union (EMU).

Regarding short-term interest rates, interest rates of the EU-11 money markets should fully converge, the latest on the first euro market day — that is, in January 1999. This requirement is expected to be reflected in the convergence of forward interest rates for settlement on 1 January 1999. In turn, the market bilateral exchange rates must converge to their central parities up to 31 December 1998. This requirement must already be resulting in a convergence between the forward exchange rate for settlement on 31 December 1998 and the respective central parity.

The announcement of the conversion rule and the need for bilateral exchange rates to converge to the announced values until 31 December 1998 restrict the paths of the spot bilateral exchange rates of the participating currencies. These market rates are expected to follow a path which becomes increasingly more insensitive to random shocks to the economic fundamentals that determine them(1). This implies that in the transition to the EMU, the correlation between variations in the EU-11 currency exchange rates vis-à-vis third currencies (say, the US dollar) shall increase.

This article presents empirical evidence on the evolution of market expectations about the Portuguese participation in the EMU, between 26 July 1996 and 30 April 1998, using information contained in the implied volatility used in the pricing of exchange rate options. We wish to test the three implications of perfect credibility:

i) the increasing correlation between the US dollar exchange rate vis-à-vis the Deutsche mark and the US dollar exchange rate vis-à-vis the escudo until the beginning of the EMU ($\rho_{t,T}$). Formally we have:

$$\rho_{t,T} \rightarrow 1 \ (T \rightarrow 0) \quad (1)$$

where $t$ is the moment in which expectations are formed and $t + T$ is 1 January 1999.

* The opinions of this paper represent the views of the authors, and are not necessarily those of the Banco de Portugal.

** Economic Research Department.

(1) See Cassola and Santos (1988) for a formal demonstration.
ii) the convergence of forward interest rates for settlement on 1 January 1999, that is:

\[ j_{t_T(T)} - j_{t_T(T)}^* = 0, \]  

(2)

where \( j_{t_T(T)} \) and \( j_{t_T(T)}^* \) are the overnight forward interest rates of the escudo and the Deutsche mark respectively, for \( t + T \) equal to 1 January 1999 (with \( t \) denoting the present moment).

iii) the equality between the forward exchange rate for settlement on 31 December 1998 and the central parity. Formally:

\[ E_i(s_{t_{T-1}}) = f_{1t_{T-1}} = s^*, \]  

(3)

where \( s_{t_{T-1}} \) stands for the logarithm of the forward bilateral exchange rate contracted in \( t \) for settlement on \( t + (T-1) \) (31 December 1998); \( E_i(.) \) is the expectations operator conditioned to the information available at moment \( t \); \( s^* \) is the logarithm of the bilateral central parity.

The remainder of this article is structured as follows: section 2 presents the methodology for calculating correlations implicit in the option premia, that will allow us to test implication (i); in the third section we show the methods for analysing the credibility of an exchange rate band using option prices, to test implication (iii); section 4 describes the data and results; the final section concludes.

2. CALCULATING EXCHANGE RATE CORRELATIONS FROM IMPLICIT VOLATILITIES

Option prices contain relevant information about market expectations of the future path of financial asset pricing and allow the estimation of risk-neutral probability density functions (RNPDF) for the underlying asset price. This issue has been addressed recently in several papers, such as Abken (1995), Bahra (1996), Neuhaus (1995) and Söderlind and Svensson (1997). Additionally, currency options permit to obtain implied exchange rate correlations, as well as to assess the market’s perception of the credibility of an exchange rate band.

Assuming no arbitrage opportunities, the exchange rate between currencies \( X \) and \( Y \) at time \( t \), denoted by \( S_{1t} \), may be written as:

\[ S_{1t} = S_{2j} \cdot S_{3j} \]  

(4)

where \( S_{2j} \) and \( S_{3j} \) are the exchange rates, respectively, between \( X \) and a third currency \( Z \) and between \( Z \) and \( Y \). Let \( s_{ij} = \ln(S_{ij}) \), with \( i = 1, 2, 3 \).

\[ s_{1t} = s_{2j} + s_{3j} \]  

(5)

Denoting the daily exchange rate variation \( s_{ij} - s_{ij-1} \) by \( r_{ij} \), we have:

\[ r_{yt} = r_{2j} + r_{3j} \]  

(6)

Let \( \sigma_{yt} \) be the standard deviation of daily exchange rate variations over a period of time from \( t \) to \( t+T \) with \( i = 1, 2, 3 \), and let \( \text{Cov}(r_{2jT}, r_{3jT}) \) be the covariance between \( r_{2j} \) and \( r_{3j} \) over the same period. The standard deviation of \( r_{yt} \) from \( t \) to \( t+T \) is given by:

\[ \sigma_{yt}^2 = \sigma_{2jT}^2 + \sigma_{3jT}^2 + 2 \cdot \text{Cov}(r_{2jT}, r_{3jT}) \]  

(7)

As \( \text{Cov}(r_{2jT}, r_{3jT}) = \rho_{yt} \sigma_{2jT} \sigma_{3jT} \) where \( \rho_{yt} \) is the correlation coefficient between \( r_{2j} \) and \( r_{3j} \) solving (7) in order to \( \rho_{yt} \) we get:

\[ \rho_{yt} = \frac{\sigma_{yt}^2 - \sigma_{2jT}^2 - \sigma_{3jT}^2}{2 \sigma_{2jT} \sigma_{3jT}} \]  

(8)

According to (8), the correlation coefficient between the daily variations of two currencies vis-à-vis a third currency may be obtained from the variance of the daily variations of the exchange rates between the three currencies (2).

There are several ways to estimate the correlation coefficient, from the information available at time \( t \). The simplest way is to compute the historical correlation over a window of \( t-T \) days. Instead of a single past correlation a moving average of several past correlations can be used as a forecast of future correlation, attaching equal or different weights to each past correlation (3).

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(2) Notice that it is irrelevant how exchange rates are expressed, since the variance of the growth rate of a variable is equal to the variance of the growth rate of its inverse.
Another way to obtain forecasts of the standard deviation is from the premia of currency put or call options. Assuming that the exchange rate \((S)\) is a log-normal variable yields:

\[
\ln S_{pt} \sim N \left[ \ln S_0 + \left( \mu - \frac{\sigma^2}{2} \right) T, \sigma \sqrt{T} \right]
\]

where \(\mu\) is the instantaneous variation of the exchange rate; \(\sigma^2\) stands for the instantaneous variance of the rate of variation of the exchange rate, being \(\sigma\) usually identified with the volatility. The variances used in calculating \(\rho_j\) in (8) are \(\sigma^2 T\).

Knowing the premia of call options \((C(X))\) and put options \((P(X))\), the implied volatility in those premia can be calculated by solving the Garman and Kohlhagen (1983) pricing formulas in order to \(\sigma\):

\[
C(X) = S e^{-jT} N(d_1) - X e^{-jT} N(d_2)
\]

\[
P(X) = X e^{-j^*T} N(-d_2) - S e^{-jT} N(-d_1)
\]

with

\[
d_1 = \frac{\ln(S/X) + (j - j^* + \sigma^2 / 2) T}{\sigma \sqrt{T}}
\]

\[
d_2 = d_1 - \sigma \sqrt{T};
\]

where \(S\) is the spot-exchange rate, \(X\) is the strike price, \(j\) is the domestic interest rate with term to maturity equal to that of the option; \(j^*\) stands for the corresponding international interest rate of the currency for which we define the option’s underlying exchange rate; \(N(d_i)\) \((i = 1, 2)\) is the value of the standardised normal distribution.

As regards over-the-counter (OTC) options, by market convention these are usually traded for a single strike price, corresponding to the forward price (i.e., they are at-the-money options).

Since implied volatilities are forward-looking, the correlations estimated this way are not affected by eventual structural breaks, as would happen with estimates from structural or time-series models. This aspect is particularly relevant in our case, since the EMU implies a change in the pattern of currency correlations.

### 3. INDICATORS OF CREDIBILITY OF AN EXCHANGE RATE BAND

The credibility of a band of fluctuation has been analysed through several indicators, namely those built from spot interest rate differentials and from spot and forward exchange rates\(^{(5)}\). In addition, information from options premia can also be used for this purpose.

#### 3.1 The risk-neutral probability density function

The simplest analysis is that consisting of deriving the risk-neutral probability density function — namely by quantifying the probability of reaching the band limits. An accessible way to obtain the risk-neutral density function from forward exchange would be to use over-the-counter prices of implied volatilities to calculate the standard deviation of the distribution\(^{(6)}\), assuming the log-normality of the exchange rate as in (9), and taking the forward exchange rate as its expected value\(^{(7)}\).

In this case equations (10) and (11) can be simplified as follows:

\[
C(X) = e^{-jT} \left[ FN(d_1) - XN(d_2) \right]
\]

\[
P(X) = e^{-j^*T} \left[ XN(-d_2) - FN(-d_1) \right]
\]

with

\[
d_1 = \frac{\ln(F/X) + (\sigma^2 / 2) T}{\sigma \sqrt{T}}
\]

\[
d_2 = d_1 - \sigma \sqrt{T}.
\]

Using OTC options also has the advantage of not forcing to the correction of the effect of getting closer to the maturity date, since the term to maturity is constant — which simplifies the interpreta-

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\(^{(3)}\) Equally weighted moving averages have the disadvantage of taking longer to reveal the impact of a shock to the market and to dissipate slowly that impact. To avoid these drawbacks, one of the most commonly used techniques are the exponentially weighted moving averages, that attribute more weight to recent observations in the calculation of standard deviations of covariances.

\(^{(4)}\) With imperfectly credible exchange rate bands, the log-normality assumption may not be the most correct one. See Malz (1996).


\(^{(6)}\) In the OTC market the volatilities (in annual percentages) are quoted instead of the options premia.

\(^{(7)}\) According to the covered interest rate parity the forward exchange rate \(F\) can be defined as \(F = Se^{(\gamma^*) T}\).
tion of the risk-neutral PDF. In the case of foreign exchange rate bands, there is still the advantage of OTC options having usually greater liquidity than those traded in the stock market\(^{(8)}\).

### 3.2 An indicator of credibility of the band

The credibility of exchange rate bands can be analysed through a more rigorous method, building indicators based on the constraints on option premia implicit in the perfect credibility assumption (see Malz (1996), Campa and Chang (1996) and Campa, Chang and Reider (1997)).

Consider a fluctuation band with upper and lower boundary denoted by \(\mathcal{S}\) and \(\underline{S}\), respectively. First, consider the extreme case of strike prices of a call option at the upper boundary of the band or above it \((X \geq \mathcal{S})\). Under perfect credibility the call option is worthless, since it will never expire in-the-money. Conversely, if the strike price of a put option is at the upper edge of the band or below it \((X \leq \underline{S})\), the put option is also worthless, since the probability of expiring in-the-money is nil.

Therefore, in the case of perfect credibility, considering the parity between the two kinds of options, the call option premium with strike price equal to or lower than \(\mathcal{S}\) is \(S_0 e^{\mathbf{j} T} - X e^{\mathbf{j} T} \) \(^{(9)}\), where \(S_0\) is the current spot exchange rate and \(\mathbf{j}\), and \(\mathbf{j}\), are respectively the foreign and domestic interest rates corresponding to the term to maturity of the option\(^{(10)/(11)}\).

For strike prices within the band the maximum value of the call option premia is given by \((\mathcal{S} - X) e^{-\mathbf{j} T}\), only reached when the exchange rate is expected to reach the band ceiling at the expiring date with certainty. This implies that perfect credibility of the band is rejected whenever the current value of the call option premium, with strike price \(X\), exceeds the maximum value that the option premium may take, assuming that the future exchange rate is within the band:

\[
C(X) > (\mathcal{S} - X)e^{-\mathbf{j} T} \Rightarrow \text{perfect credibility rejection} \quad (14)
\]

The other constraint on options premia in a credible exchange rate band is that resulting from the convexity relationship between the option premium and the strike price. The argument can be exposed in two steps. First, each unitary increase in the strike price yields a maximum reduction of the value of the call options expiring in-the-money that equals the current value of that unit (when the probability is 1). Second, the higher the strike price, the less probable that the call option expires in-the-money, and hence the smaller the reduction in the call option value for each unitary increase in the strike price. Thus we have:

\[
-e^{-\mathbf{j} T} \leq \frac{\partial C(X)}{\partial X} \leq 0 \quad e^{-\mathbf{j} T} \frac{\partial^2 C(X)}{\partial X^2} \geq 0.
\]

Diagram 1 illustrates the constraints on the behaviour of options prices in a perfectly credible exchange rate band. Straight line (1) represents the line of maximum slope that passes through \(C(\mathcal{S})\) (i.e., the call option premium with strike price \(\mathcal{S}\)), when the unit rise in the strike price cuts the option premium by the present value of that unit. This slope is the symmetric of the discount factor \((-e^{-\mathbf{j} T}\)\(^{(12)}\). Straight line (2) joins the prices of the options with strike prices \(\underline{S} \leq \mathcal{S}\), being \(C(\underline{S})\) and \(C(\mathcal{S})\), respectively equal to zero and \(S_0 e^{\mathbf{j} T} - \underline{S} e^{-\mathbf{j} T}\) \(^{(13)}\).

Since the relationship between the option price and the strike price is strictly convex, the call option price should stand in a convex curve between (1) and (2), that contains points \((\mathcal{S}, C(\mathcal{S}))\) and for

\[\text{(10) Recall that the parity between the call and the put options establishes that } P = C - (F - X)c^{-\mathbf{j} T}, \text{ where } F \text{ is the forward exchange rate and } P = 0 \text{ whenever } X \leq \underline{S}.\]

\[\text{(11) Using capitalised interest in continuous time, Campa and Chang (1996) consider discrete capitalisation.}\]

\[\text{(12) The straight line crosses point } F, \text{ since its slope equals } \frac{-C(\underline{S})}{\underline{S} - \underline{S}}. \text{ with } x \text{ denoting the horizontal intercept. Therefore, equalling that ratio to the value of the slope of line } (-e^{-\mathbf{j} T}) \text{ yields } x = F.\]

\[\text{(13) The value of } C(\mathcal{S}) \text{ is found by substituting } \underline{S} \text{ in the expression for the call option premium with strike price equal to or lower than } \underline{S}, \text{ referred above.}\]

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any strike price ranging between $S$ and $\bar{S}$. Therefore, the perfect credibility of an exchange rate band can always be rejected provided that for a given point in the call option price function (for $X < \bar{S}$) the absolute value of the slope joining that point to $S$ is greater than the slope of line (2). This being the case, given that the call option price function is strictly convex, the price of a call option with strike price greater than $\bar{S}$ is positive, which means that the probability of the exchange rate exceeds the band upper bound is not nil.

The rejection of perfect credibility thus corresponds to checking if the following convexity condition holds:(14)

$$C(X) > \frac{\bar{S} - X}{\bar{S} - S} \left( S_0 e^{-\eta T} - \bar{S} e^{-\eta T} \right)$$

⇒ Perfect credibility rejection.

As mentioned in Campa and Chang (1996), condition (15) is more restrictive than condition (14) anytime the forward rate is within the band.(15)

In the case of wide exchange rate bands — as happens currently with the ERM of the EMS — the results should be expected to point in general to the non-rejection of perfect credibility, so the exercise provides no additional relevant information. However, more interesting exercises can be attempted in these cases — e.g. using (15) to calculate the smaller possible bandwidth that would permit the non-rejection of its perfect credibility — i.e., the minimum size of a perfectly credible band ($\alpha$).

Bearing in mind the perspectives of monetary integration in the EU, this exercise allows to use options prices to identify the band in which a currency could float without obliging to a realignment within the term to maturity of the option. With the beginning of the EMU becoming closer, the currency would then be expected to float in a progressively narrower interval.

Transforming (15) into an identity and using the principle of a band’s symmetry around the central parity yields $\bar{S} = \frac{S_c}{(1 + \alpha)}$ and $\bar{S} = (1 + \alpha)S_c$ (where $S_c$ is the central parity). Substituting $\frac{S}{S}$ and $\bar{S}$ for these expressions, the equation resulting from (15) can be solved in order to $\alpha$, giving $S_c$.

4. EMPIRICAL EVIDENCE

The data consisted in volatilities (ask quotes) of OTC at-the-money 3-month forward options, disclosed by the Banco Português do Atlântico (BPA)(16), for the exchange rate of German Mark/Escudo (PTE/DEM), German Mark/US Dollar (USD/DEM) and US Dollar/Escudo (PTE/USD) for the period between 26 July 1996 and 30 April 1998(17).

(14)The slope, at a given point $X$ of the straight line linking it to $(\bar{S},0)$ equals $\frac{C(S)(\bar{S} - S)}{\bar{S} - S}$. To reject perfect credibility of an exchange rate bond, that slope should exceed in absolute value that of line (2) (equal to $\frac{C(S)}{\bar{S} - S}$). That is,

$$C(X) > \frac{\bar{S} - X}{\bar{S} - S} \cdot \left( S_0 e^{-\eta T} - \bar{S} e^{-\eta T} \right)$$

⇒ Perfect credibility rejection.

(15)Multiplying and dividing the right-hand term of (15) by $e^{\eta T}$ yields:

$$C(X) > \frac{\bar{S} - X}{\bar{S} - S} \cdot e^{\eta T} \cdot \left( S_0 e^{-\eta T} - \bar{S} e^{-\eta T} \right)$$

⇒ $C(X) > \frac{\bar{S} - X}{\bar{S} - S} \cdot \bar{S} e^{-\eta T} \cdot (F - \bar{S})$ ⇒ $C(X) > \left( \frac{\bar{S} - X}{\bar{S} - S} \right) e^{-\eta T} \left( \frac{F - \bar{S}}{\bar{S} - \bar{S}} \right)$

The first factor of the right-hand term in this inequality is the right-hand term of inequality (14), so (15) is a more restrictive condition when the second factor is smaller than 1 (i.e., when $\bar{S} < F < \bar{S}$).
We started by analysing implication (i) \( \rho_{rT} \rightarrow 1 \) \( (T \rightarrow 0) \), calculating the implicit correlation of the escudo and Deutsche mark exchange rates vis-à-vis the US dollar according to (8) and considering \( S_1, S_2 \) and \( S_3 \) as the PTE/DEM, USD/DEM and PTE/USD exchange rates, respectively (18).

According to chart 1, the correlation between the German Mark and the Portuguese Escudo exchange rates, vis-à-vis the US Dollar, has been increasing since the end of the first quarter of 1997. In the second half of 1997, the implied correlation was consistently above 0.95 and rose significantly during September. The international stock market correction in October 1997 did not affect the implied correlation, which was relatively stable between 0.98 and 0.99.

In March 1998, following the announcement of the 1997 fiscal deficits of EU countries, the implied correlation increased to above 0.99. On the eve of the Brussels summit of 1-3 May, the correlation between the exchange rate variations of DEM/USD and USD/PTE was already between 0.99 and 1.

Therefore, the results from implied correlations are consistent with the general assertion that financial market participants anticipated the inclusion of Portugal in the group of the Euro founding members.

The relationship between the behaviour of implied correlations and the expectations on Escudo’s participation in the Euro-area seems to be confirmed by the analysis of implication (ii). Indeed, exchange rate correlations between the daily rates of change of the Portuguese Escudo and the German Mark rates vis-à-vis the US dollar and the spread between the Portuguese Escudo and the German Mark forward overnight rate, for settlement in 1 January 1999, show a trend which is consistent with the anticipation of the Portuguese participation (Chart 1).

As regards implication (iii),

\[ E \left( S_{t+nT} \right) = f_{\mu(t+nT)} = s' \],

as charts 2 and 3 show, the mean, mode and median of the RNPDF of the PTE/DEM exchange rate derived according to (9) converged to the bilateral central parity (102.505 PTE/DEM); moreover, the probability attributed to the values in the edges of the distribution decreased. The interquartile interval decreased from PTE 2 to PTE 0.9.

The anticipation of the Portuguese participation in the EMU is also evident in the progressive reduction of the minimum width of the perfectly credible band vis-à-vis the Deutsche mark — which corresponds to the solution of (15) solved in order to \( \alpha \). According to chart 4, this variable fell sharply since July 1996, from values over 3 per cent to around 0.5 per cent. This behaviour is consistent with the shift in expectations towards a lower depreciation of the escudo vis-à-vis the Deutsche mark, but also with reduction in market uncertainty about the future values of PTE/DEM.

5. CONCLUSIONS

Currency option prices provide relevant information concerning expectations of economic agents. Comparing to the usual correlation measures conditional to past observations, correlations implied in exchange rate option prices offer the advantage of being forward-looking, and are par-
particularly useful in periods in which regime shifts are expected. Considering EMU, implied correlations permit to assess market expectations on the degree of convergence between the participating currencies.

The results we obtained are consistent with the idea that the probability of Escudo’s participation in the Euro-area from the beginning increased persistently over the course of 1997, and that the market attributed a high degree of credibility to the bilateral central parity announced at the 1-3 May 1998 European summit.

On the eve of the summit, market participants were expecting a perfect correlation between the movements of the Deutsche mark and the Portuguese Escudo, vis-à-vis the US Dollar. Furthermore, the central moments of the PTE/DEM exchange rate RNPDF converged to the central parity and the uncertainty regarding the future values of PTE/DEM decreased strongly.

REFERENCES


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